Synthesis of Mechanism

Study of motion

- **Kinematic Analysis** (Study displacement, velocity & acceleration characteristics of mechanism)
- **Kinematic Synthesis** (Synthesis of mechanism in the design or creation of a mechanism to produce a required output motion for a given input motion.)

Examples:-

1. Determination of no. of tooth in gear to obtain required velocity ratio
2. Designing cam to give follower a known motion.

Synthesis Process :-

Steps in Synthesis of mechanism process.

Synthesis

Type synthesis

- First step in kinematic synthesis
- Selection of mechanism
- type of links
- aspects like mtol, manufacturing process, space should be consider

Number synthesis

- Second step
- No. of links, joints
- No. of D.O.F hence Grubler's equation is used.

Dimensional Synthesis.

- Third step
- Dimensions of mechanism
- length of link, distance between pivot, type links (binary or ternary) angle, cam, gear ratio, cam contours etc.

* Tasks of Kinematic Synthesis

Three - tasks :-

(1) Function generation

(2) Path generation

(3) Motion generation (Body guidance)

(1) Function generation

The general requirement is that the output link should rotate, oscillate or reciprocate according to specified function of time or function of Input motion.

For e.g. $Y = F(x)$

$x \rightarrow$ motion of Input link
$y \rightarrow$ — " — of output — " —

(2) Path generation

The mechanism is required to guide a point (tracer point or coupler point) along a path having a prescribed shape.

Common requirement : circular arc, elliptical, straight line

(3) Motion generation (Body guidance)

Entire body guided through prescribed motion. Both position of a point within a moving body and angular displacement of body are specified.

Simple translation or combination of translation & rotation

* Synthesis of function Generation

A frequent requirement in design is to cause an output link to rotate, reciprocate or oscillate according to a specified function of time or motion of input link. This is called function generation

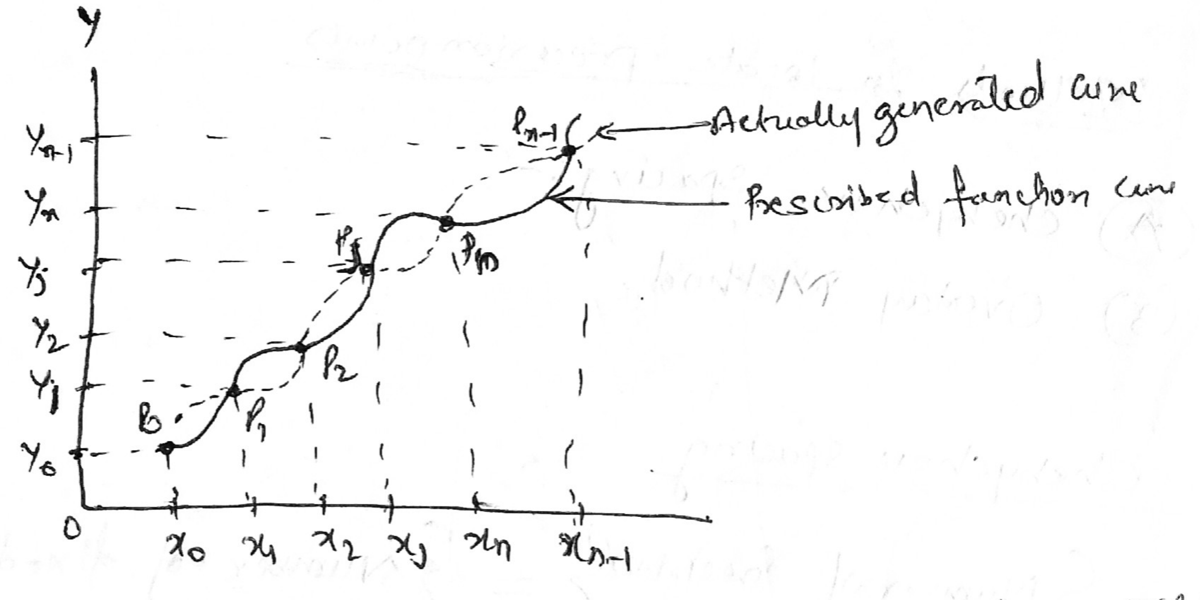
$$Y = f(x)$$

$x \rightarrow$ Angular Position of Input link

$Y \rightarrow$ — " — " — " — o/p — " —

* (1) Precision Positions :-

While designing a mechanism to generate a particular function it is usually not possible to produce mathematically exact solution.



$P_0, P_1, P_2, P_3, P_n, P_{n-1}$ are the precision points or precision positions or accuracy points.

This are the points where it satisfies with required function.

② Structural Error & Mechanical Error

- Difference between required motion & actual motion generated is called as structural error.
- ~~The~~ Similarly there are manufacturing Problems ~~error~~ also.
- The error produced from tolerences in length of links & bearing clearances is called as mechanical error.
- Structural errors are present even if there is no graphical or mechanical error.
- The amount of structural error in the soln can be affected by selection of precision points.

Methods to locate precision points

(A) Chebychev Spacing :-
(B) Overlay Method.

(A) Chebychev spacing

- {Number of Precision Position} = {Number of fixed Parameters that may be used in design}

It generally varies between 3 to ~~5~~ 6.

- The best spacing of the precision points, for the first trial is known as Chebychev spacing.

As per Chebyshev Spacing for n points in range $x_s \le x \le x_F$

$$x_j = \frac{1}{2}(x_F + x_s) - \frac{1}{2}(x_F - x_s)\cos\left(\frac{\pi(2j-1)}{2n}\right)$$

or,

$$x_j = \frac{1}{2}(x_F + x_s) - \frac{1}{2}\Delta x \cos\left[\frac{\pi(2j-1)}{2n}\right]$$

where,

x_j = Precision Points

x_s = Starting Position

x_F = Finishing Position

Δx = Range in x = $x_F - x_s$

$j = 1, 2, 3 \ldots n$

n = No. of precession Points.

For example,

Synthesize a linkage mechanism to generate the function $y = x^{1.1}$ over range $1 \le x \le 3$ by using 3 precision points.

Ans $x_s = 1$, $x_F = 3$, $\Delta x = x_F - x_s = 3 - 1$

$\Delta x = 2$

So, $n = 3$ so, $j = 1, 2, 3$

$$x_j = \frac{1}{2}(x_F + x_s) - \frac{1}{2}\Delta x \cos\left(\frac{\pi(2j-1)}{2n}\right)$$

$$x_1 = \frac{1}{2}(3+1) - \frac{1}{2}(2)\cos\left(\frac{\pi[2(1)-1]}{2(3)}\right)$$

$$= \frac{1}{2}(4) - \cos\left(\frac{\pi}{6}\right)$$

$$= 2 - 0.866025$$

$$x_1 = 1.133.$$

$$x_2 = \frac{1}{2}(3+1) - \frac{1}{2}(2)\cos\left[\frac{\pi(2(2)-1)}{2\times 3}\right]$$

$$= \frac{1}{2}(4) - \cos\left[\frac{3\pi}{6}\right]$$

$$= 2 - 0$$

$$\boxed{x_2 = 2}$$

$$x_3 = \frac{1}{2}(3+1) - \frac{1}{2}(2)\cos\left[\frac{\pi(2(3)-1)}{2(3)}\right]$$

$$= \frac{1}{2}(4) - \cos\left[\frac{5\pi}{6}\right]$$

$$= 2 - (-0.866602)$$

$$\boxed{x_3 = 2.866}$$

Substituting in function eqn $y = x^{1.1}$

$$y_1 = (1.1333)^{1.1} = 1.147$$

$$\boxed{y_1 = 1.147}$$

$$y_2 = (2)^{1.1} = 2.143$$

$$\boxed{y_2 = 2.143}$$

$$y_3 = (2.866)^{1.1}$$

$$\boxed{y_3 = 3.184}$$

Hence precision position are,

$x_1 = 1.133$, $y_1 = 1.147$

$x_2 = 2$, $y_2 = 2.143$

$x_3 = 2.866$, $y_3 = 3.184$

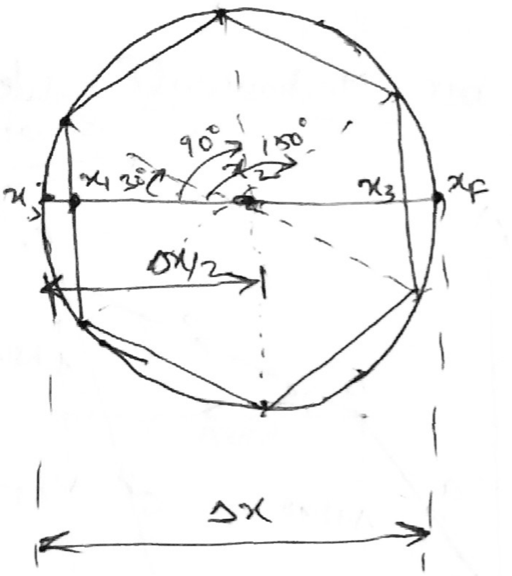
The three Position can also be find out by graphical method.

radius of circle $= x_F - x_s = \Delta x = 2$

Polygon ~~size~~ Sides = 2 × (no. of Precision points)

$= 2 \times n$

$= 2 \times 3 = 6$



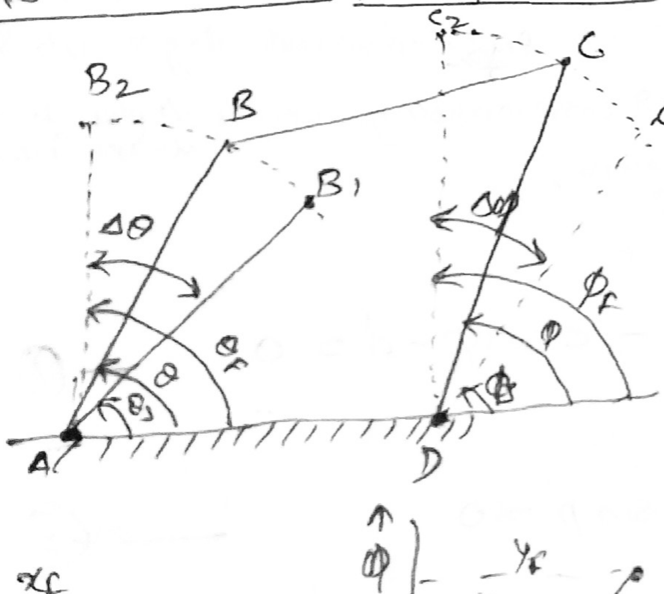
Draw line from each corner of polygon on the diagonal of circle

* Angle Relationship for function generation

Function of Mechanism,

$y = f(x)$

over a range of $\Delta\theta, \Delta\phi, \Delta x, \Delta y$



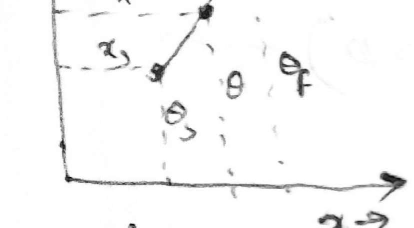
then,

$\Delta\theta = \theta_F - \theta_s$

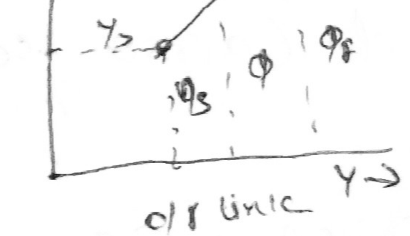
$\Delta\phi = \phi_F - \phi_s$

$\Delta x = x_F - x_s$

$\Delta y = y_F - y_s$



I/P link



O/P link

So from relation betw x & θ, y & ϕ we can say,

$$\frac{\Delta\theta}{\Delta x} = \frac{\theta_F - \theta_s}{x_F - x_s}$$

So,

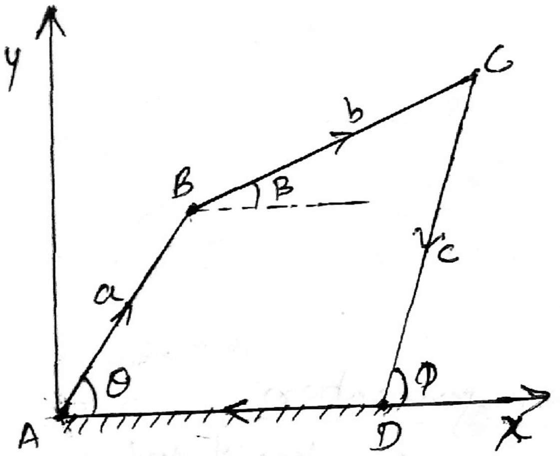
$$\theta_f = \frac{\Delta\theta}{\Delta x}(x_f - x_s) + \theta_s$$

$$\theta_j = \frac{\Delta\theta}{\Delta x}(x_j - x_s) + \theta_s$$

Similarly,

$$\phi_j = \frac{\Delta\phi}{\Delta y}(y_j - y_s) + \phi_s$$

* Analytical Synthesis of 4-bar Mechanism (Freudenstein's Equation)



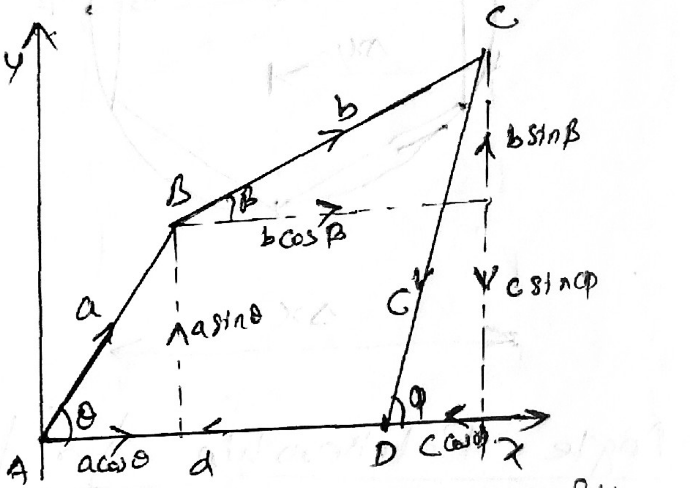


Fig :- 4-bar Mechanism

Fig:- Components along x-axis & y-axis

considering links as vector & corresponding vector displacement along x-axis & y-axis for equlbm of 4-bar mechanism,

along x-axis

$$a\cos\theta + b\cos\beta - c\cos\phi - d = 0 \quad \text{--- (1)}$$

along y-axis

$$a\sin\theta + b\sin\beta - c\sin\phi = 0 \quad \text{--- (2)}$$

from eqution (1)

$$b\cos\beta = d - (a\cos\theta - c\cos\phi) \quad \text{--- (3)}$$

from eqution (2)

$$b\sin\beta = c\sin\phi - a\sin\theta \quad \text{--- (4)}$$

Squaring & adding both eqn ③ & ④

$$b^2\cos^2\beta + b^2\sin^2\beta = \{d - (a\cos\theta - c\cos\phi)\}^2 + \{c\sin\phi - a\sin\theta\}^2$$

$$b^2 = \{d^2 + (a\cos\theta - c\cos\phi)^2 - 2d\{a\cos\theta - c\cos\phi)\}$$
$$+\{c^2\sin^2\phi + a^2\sin^2\theta - 2ac\sin\phi\sin\theta\}$$

$$b^2 = d^2 + a^2\cos^2\theta + c^2\cos^2\phi - 2ac\cos\theta\cdot\cos\phi - 2ad\cos\theta$$
$$+ 2dc\cos\phi + c^2\sin^2\phi + a^2\sin^2\theta - 2ac\sin\phi\sin\theta$$

$$= d^2 + a^2[\cos^2\theta + \sin^2\theta] + c^2[\cos^2\phi + \sin^2\phi]$$
$$- 2ac(\cos\theta\cdot\cos\phi + \sin\theta\sin\phi] - 2ad\cos\theta + 2dc\cos\phi$$

$$b^2 = d^2 + a^2 + c^2 - 2ac\cos(\theta - \phi) - 2ad\cos\theta + 2dc\cos\phi$$

$$2ac\cos(\theta - \phi) = a^2 - b^2 + c^2 + d^2 - 2ad\cos\theta + 2dc\cos\phi$$

$$\cos(\theta - \phi) = \frac{a^2 - b^2 + c^2 + d^2}{2ac} - \frac{d}{c}\cos\theta + \frac{d}{a}\cos\phi$$

$$\cos(\theta - \phi) = \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta + \frac{(a^2 - b^2 + c^2 + d^2)}{2a}$$

$$\boxed{\cos(\theta - \phi) = K_1\cos\phi - K_2\cos\theta + K_3}$$

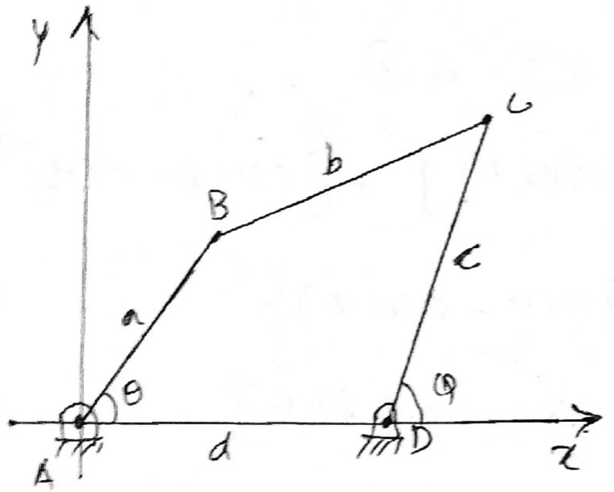
where,

$$K_1 = \frac{d}{a} \quad \text{and} \quad K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2a}.$$

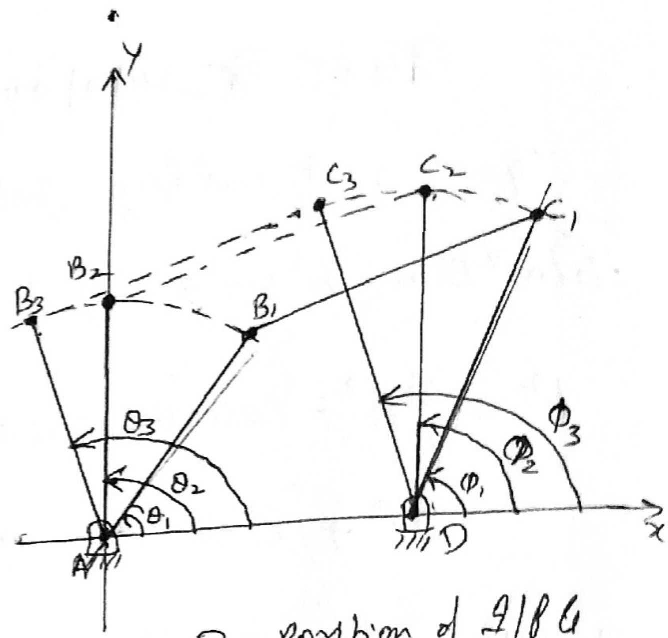
$$K_2 = \frac{d}{c}$$

The above equation is called Freudenstein's Equation.

Three position Synthesis



4-Bar mechanism



3-position of I/P & O/P links.

let three angular position of input link (AB) = $\theta_1, \theta_2, \theta_3$

— " — " — " — output — " — (CD) = ϕ_1, ϕ_2, ϕ_3

In order to find the dimensions of 4-bar Mechanism

By using Freudenstein's Equation,

$$\cos(\theta_1 - \phi_1) = K_1 \cos\phi_1 - K_2 \cos\theta_1 + K_3$$

$$\cos(\theta_2 - \phi_2) = K_1 \cos\phi_2 - K_2 \cos\theta_2 + K_3$$

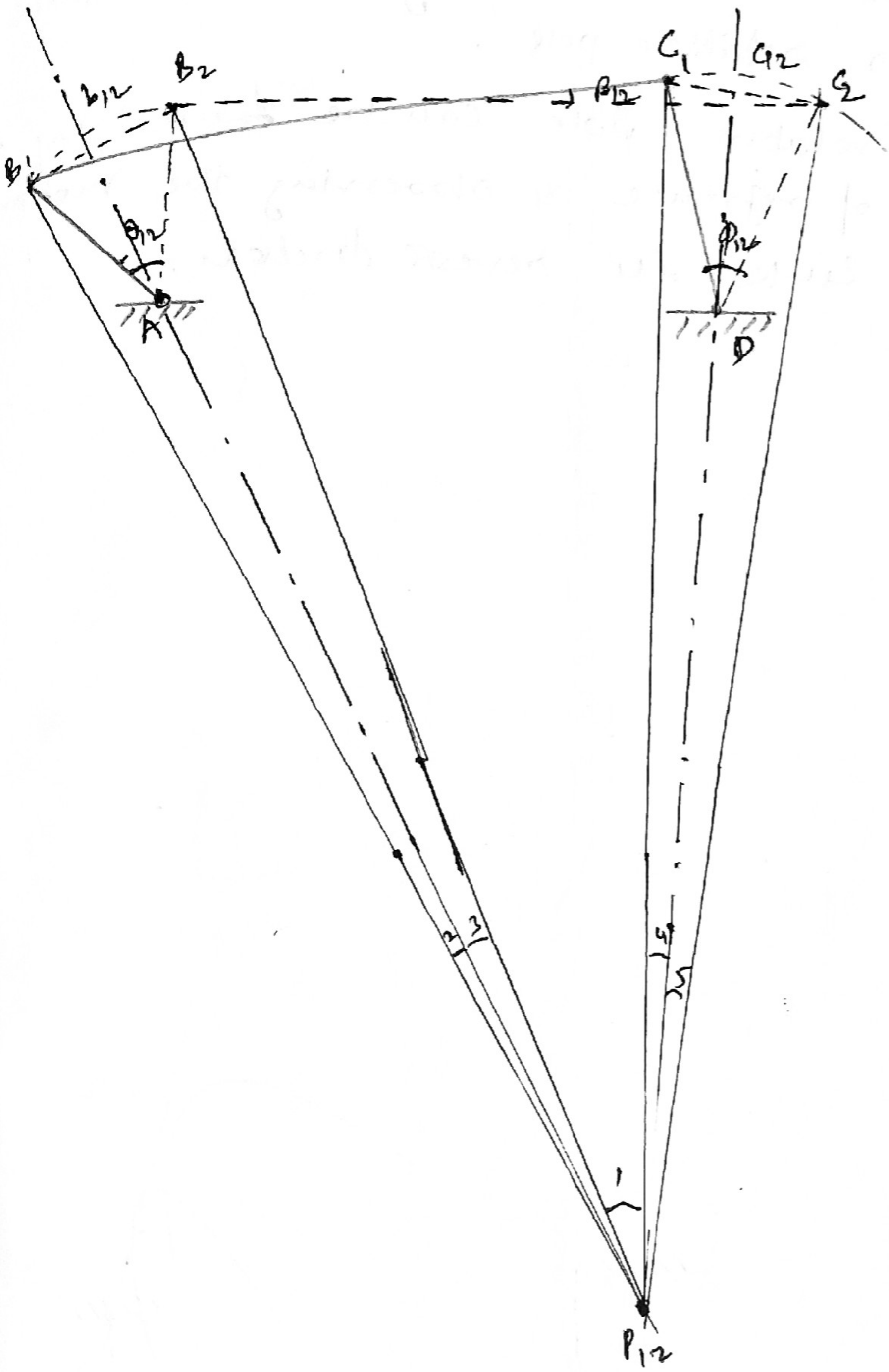
$$\cos(\theta_3 - \phi_3) = K_1 \cos\phi_3 - K_2 \cos\theta_3 + K_3.$$

By solving above equation we can find K_1, K_2, K_3

and from K_1, K_2, K_3 we can find a, b, c, d.

Pole (Graphical Method)

A pole of moving link is the centre of its rotation w.r.t fixed line.



① $AB_1 = AB_2$

② b_{12} is midnormal of B_1B_2 passing through A

c_{12} —"—"— C_1C_2 —"—"— D

③ $\Delta B_1P_{12}C_1 \cong \Delta B_2P_{12}C_2$

$\angle 2 + \angle 3 + \angle 1 = \angle 1 + \angle 4 + \angle 5$

④ $\angle 2 + \angle 3 = \angle 4 + \angle 5$

⑤ $\angle 2 = \angle 3$ and $\angle 4 = \angle 5$ ⑥ $\beta_{12} = \angle 4 + \angle 5 = \angle 2 + \angle 3$

Basic Terminology...

* **Relative pole**

when the rotation of link is considered relative to another moving link the pole is known as relative pole.

relative pole can be found by fixing the link of refrence & observing the motion of the other link in reverse direction.

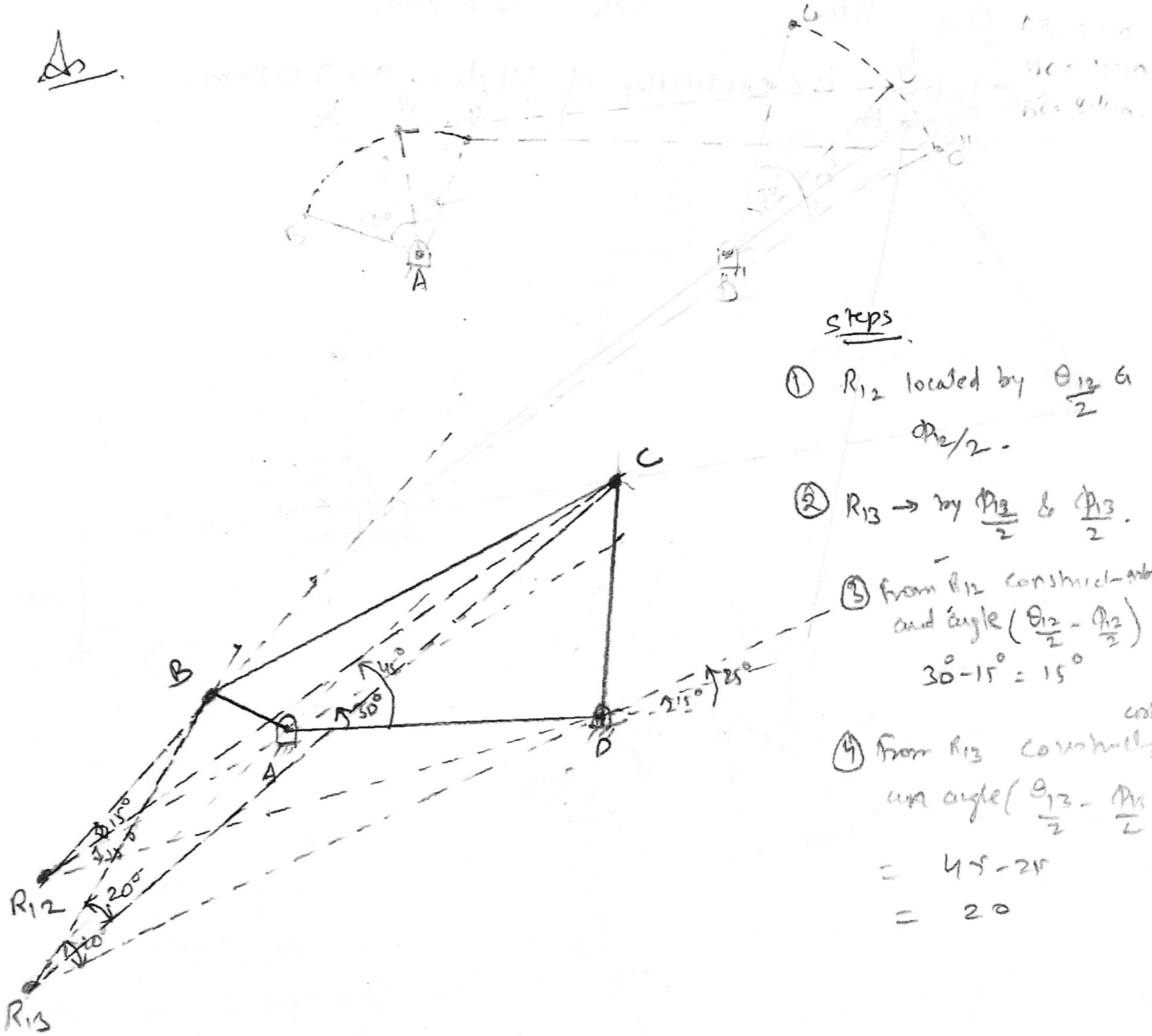
Relative pole method

Q1 Design a 4-link mechanism to coordinate 3 positions of input & the output links for following angular displacements

$\theta_{12} = 60^\circ$ $\phi_{12} = 30^\circ$

$\theta_{13} = 90^\circ$ $\phi_{13} = 50^\circ$

Ans.



steps

1. R_{12} located by $\frac{\theta_{12}}{2}$ & $\phi_{12}/2$.

2. R_{13} → by $\frac{\theta_{13}}{2}$ & $\frac{\phi_{13}}{2}$.

3. from R_{12} construct arbitrary and angle $\left(\frac{\theta_{12}}{2} - \frac{\phi_{12}}{2}\right)$

   $30^\circ - 15^\circ = 15^\circ$

4. From R_{13} construct an angle $\left(\frac{\theta_{13}}{2} - \frac{\phi_{13}}{2}\right)$

   $= 45 - 25$

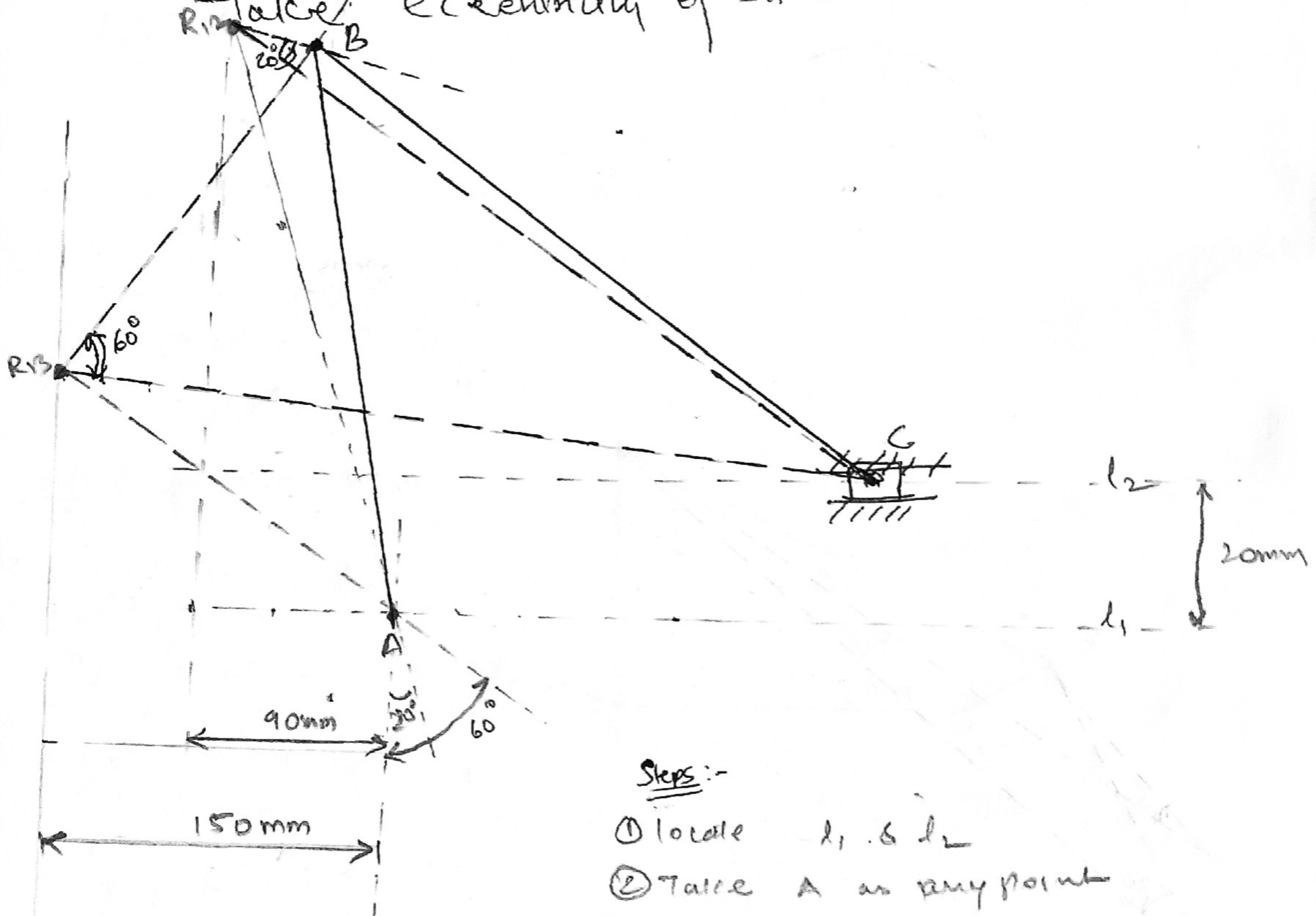
   $= 20$

Qn: Design a slider crank mechanism to coordinate 3 positions of the input link & the slider for the following angular & linear displacement of input link & slider respectively

$\theta_{12} = 40^\circ$ $S_{12} = 180$ mm

$\theta_{13} = 120^\circ$ $S_{13} = 300$ mm.

Take eccentricity of slider as 20mm.



Steps:-

① locate l_1 & l_2

② Take A as any point

③ locate R_{12} by rotating vertical line by $\frac{\theta_{12}}{2} = 20^\circ$ & vertical line at $\frac{S_{12}}{2} = 90$mm.

④ locate R_{13} by rotating vertical line by $\frac{\theta_{13}}{2} = 60^\circ$ & a vertical line at $\frac{S_{13}}{2} = 150$ mm.

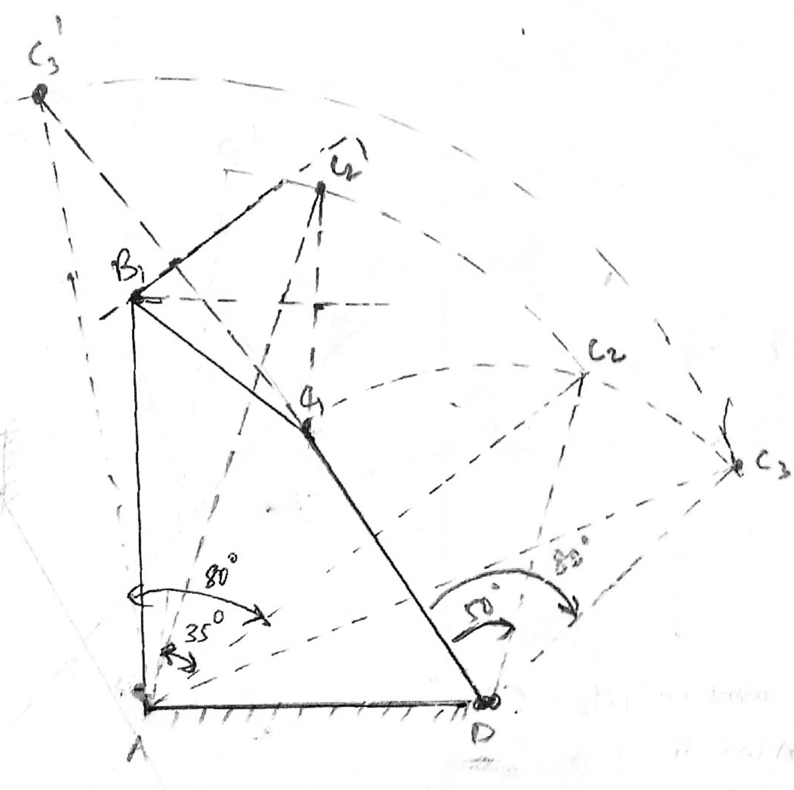
⑤ from R_{12} & R_{13} draw an angle of 20° & 60° respectively so that intersection of this angles will give points B & C.

* Inversion Method

Que Design a four-link Mechanism to co-ordinate a 3 positions of input & of the output links for the following angular displacements by inversion method. $\theta_{12} = 35^\circ$, $\phi_{12} = 50^\circ$

$\theta_{13} = 80^\circ$, $\phi_{13} = 80^\circ$

Ans.



Steps

① Take AD fix line.

② draw DC_1 i.e first position of output link arbitrarily

③ Draw 50°, 80° to get position DC_2 & DC_3 respectively

④ Connect AC_2 & AC_3 and draw angles 35° & 80° respectively to get point C_2' & C_3' respectively.

⑤ Connect C_2' & C_1 and C_3' & C_1

⑥ Draw midnormal between C_1C_3' and C_1C_2'

⑦ The Intersection of midnormal of C_1C_3' and C_1C_2' will give the B_1 point

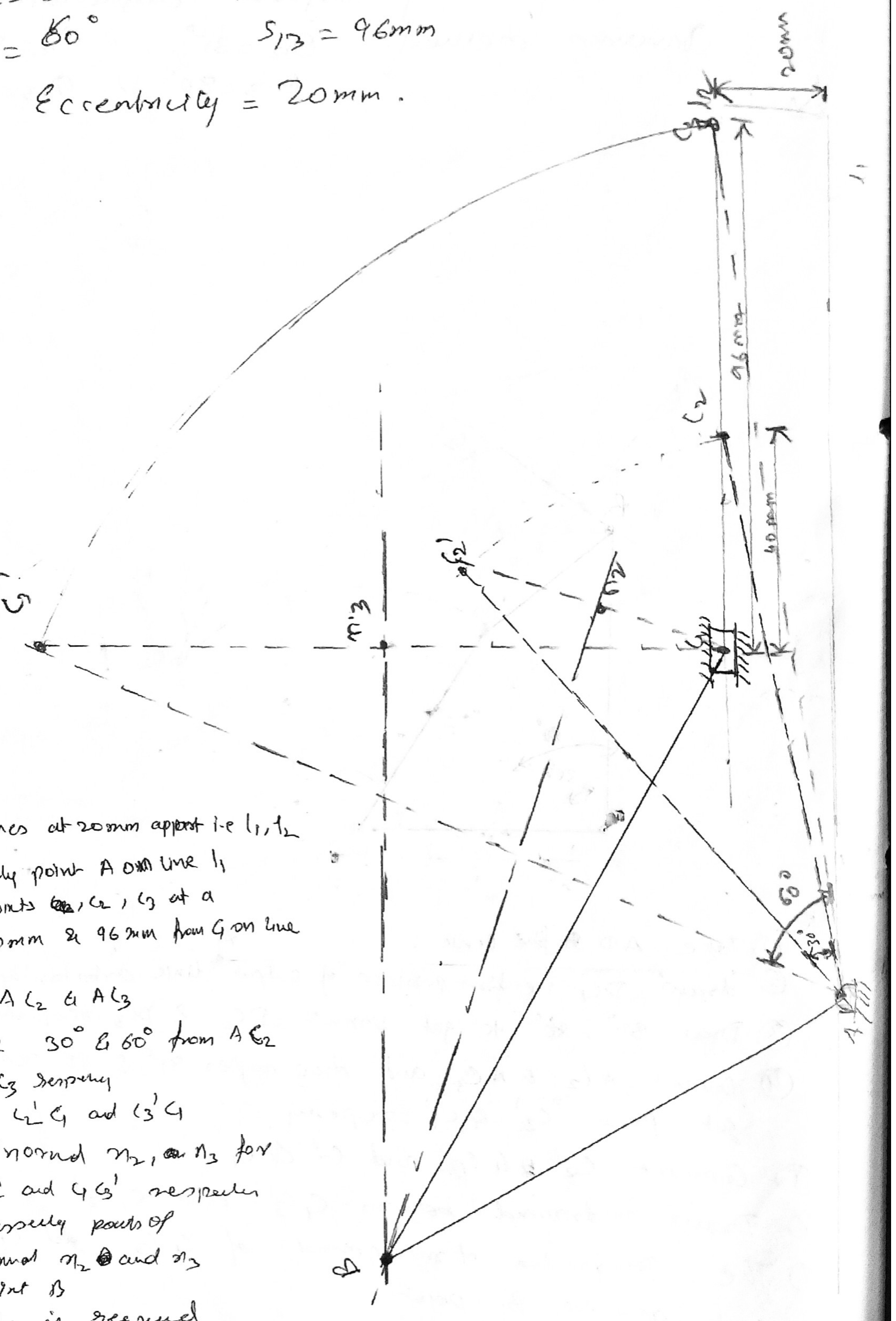
⑧ Connect AB_1 and C_1B_1 hence AB_1C_1D is required 4-bar mechanism.

Q. Design a slider-crank mechanism to coordinate 3 position of the input and slider for the following data by inversion method.

$\theta_{12} = 30^\circ$ $S_{12} = 40mm$

$\theta_{13} = 60^\circ$ $S_{13} = 96mm$

Eccentricity = 20mm.



Steps:-

1. Draw two lines at 20mm appart i.e l_1, l_2
2. Take arbitrarily point A on line l_1
3. Take points C_1, C_2, C_3 at a distance 40mm & 96 mm from C_1 on line l_2
4. Connect AC_2 & AC_3
5. draw angle 30° & 60° from AC_2 and AC_3 respectively
6. Connect $C_2'C_1$ and $C_3'C_1$
7. Draw midnormal n_2, n_3 for line C_1C_2' and C_1C_3' respectively
8. The intersecting points of midnormal n_2 and n_3 is point B
9. ABC_1 is required slider crank mechanism.